Fractions

Description of Reasoning Strategies

Following are the specific reasoning strategies and understandings that relate to Fractions Reasoning strategies are organized into two categories—those appropriate and those not appropriate to the numbers at hand in the problem.

Comparing Mentally

Strategies appropriate to the numbers at hand

- **Uses relationships between numerators and denominators to compare**
  Understanding relationships between numerators and denominators can make comparisons with fractions easier. For example, when comparing $\frac{3}{8}$ and $\frac{5}{6}$ some students reason that $\frac{5}{6}$ is closer to 1, or that $\frac{3}{8}$ is less than $\frac{1}{2}$, and $\frac{5}{6}$ is greater. When comparing $\frac{5}{12}$ and $\frac{5}{8}$, some students reason that eighths are larger than twelfths and since there are the same number of each, $\frac{5}{8}$ must be greater than $\frac{5}{12}$.

Strategies not appropriate for the numbers at hand

- **Converts to common denominators to compare**
  Converting to common denominators is not always an efficient strategy for comparing fractions. For example, when mentally comparing $\frac{3}{8}$ and $\frac{5}{6}$, converting to common denominators indicates a lack of attention to the relationships between the numerators and denominators of the fractions at hand.

Computing Mentally

Strategies appropriate to the numbers at hand

- **Reasons with decimals or percents**
  When students reason with decimals or percents to compare and compute with fractions, they show an understanding of equivalence. Quick recall of common equivalents can make mental comparison and computation easier. For example, when solving $3\frac{1}{2} \times 2$ students may relate the problem to decimals and reason that $3.5 \times 2 = 7$.

- **Extends understanding of operations with whole numbers to operations with fractions**
  Applying understanding of operations with whole numbers to operations with fractions is essential for computing efficiently with fractions. For example, when solving $3\frac{1}{2} \times 2$, some students apply the distributive property by multiplying 3×2 to get 6, $\frac{1}{2} \times 2$ to get 1, and then adding $6 + 1$. Other students apply the understanding of multiplication as adding equal groups and think of $3\frac{1}{2} \times 2$ as $3\frac{1}{2} + 3\frac{1}{2}$.

- **Uses benchmark of $\frac{1}{2}$ or 1 to estimate**
  Learning to estimate is as important as learning to perform exact calculations. Estimation can be used to check the reasonableness of answers or to figure out an answer that does not need to be exact. The strategy of using benchmark numbers to make estimates requires relating fractions to an appropriate benchmark and then computing mentally. For example, when asked whether $\frac{11}{12} + \frac{1}{5}$ is greater than 1 or less than 1, some students reason that $\frac{11}{12}$ is $\frac{1}{12}$ away from 1 and $\frac{1}{5}$ is greater than $\frac{1}{12}$ so the sum must be greater than 1.

Strategies not appropriate for the numbers at hand

- **Uses standard algorithm to compute**
  While using a standard algorithm is not a concern for an individual problem, it's a concern when students rely on the algorithm as their only strategy for computing mentally. For example, figuring out $3\frac{1}{2} \times 2$ by renaming $3\frac{1}{2}$ to $\frac{7}{2}$ and then using the multiplication algorithm is inefficient and may indicate a lack of being able to numerically reason.

- **Figures exact answer to estimate**
  Learning to estimate is as important as learning to perform exact calculations. Estimation can be used to check the reasonableness of answers or to figure out an answer that does not need to be exact. Relying on figuring exact answers when estimating may indicate a lack of flexibility to reason in other ways with the numbers at hand. For example, when asked if $\frac{11}{12} + \frac{1}{5}$ is greater or less than 1, students who need to compute the exact answer indicate a lack of being able to reason numerically.

Applying Understanding

- **Models with mathematics to solve problems in context**
  Solving problems in contexts requires that students can model situations mathematically—relate situations to the appropriate numerical operations and provide answers that relate to the problem contexts. For example, to figure out how many $\frac{1}{4}$-pound hamburgers can be made from $2\frac{1}{2}$ pounds of meat, students figure out the number of $\frac{1}{4}$s in $2\frac{1}{2}$ by either reasoning with equivalent fractions ($2\frac{1}{2} = \frac{10}{4}$) or dividing $2\frac{1}{2}$ by $\frac{1}{4}$.

- **Understands equivalence in context**
  Students may be able to apply a procedure to convert a fraction like $\frac{3}{4}$ into $\frac{6}{8}$. However, it's also important for students to understand the meaning of equivalence; that is, that the two fractions represent the same quantity. For example, when told that Carlos lives $\frac{3}{4}$ of a mile from school and Terrell lives $\frac{6}{8}$ of a mile from school, students who understand equivalence know that the boys lives the same distance from school.